

7.1 Trigonometric Identities and Equations

- **Identities Versus Solving Equations**

Identities are equations that are true for all values of the variable or for a large set that is specified already. Equations that we solve are only true for certain values (the set of their solutions).

- **How to Verify Identities:** Use the following identities and simplify each side separately to equivalent functions or start with Left Hand Side and simplify to get to the Right Hand Side or the other way around.

- **Pythagorean identities:**

$$\sin^2(t) + \cos^2(t) = 1$$

$$\sec^2(t) = \tan^2(t) + 1$$

$$\csc^2(t) = 1 + \cot^2(t)$$

- **Even and Odd:**

Odd:

$$\sin(-t) = -\sin(t)$$

$$\tan(-t) = -\tan(t)$$

$$\cot(-t) = -\cot(t)$$

$$\csc(-t) = -\csc(t)$$

Even:

$$\cos(-t) = \cos(t)$$

$$\sec(-t) = \sec(t)$$

- **Reciprocal identities**

$$\tan(t) = \frac{1}{\cot(t)} \quad \text{when } \cot(t) \neq 0$$

$$\cot(t) = \frac{1}{\tan(t)} \quad \text{when } \tan(t) \neq 0$$

$$\sin(t) = \frac{1}{\csc(t)} \quad \text{when } \csc(t) \text{ is defined}$$

$$\cos(t) = \frac{1}{\sec(t)} \quad \text{when } \sec(t) \text{ is defined}$$

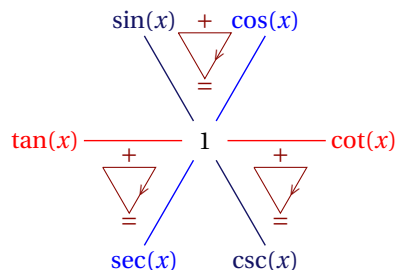
- **Quotient identities**

$$\tan(t) = \frac{\sin(t)}{\cos(t)} \quad \cos(t) \neq 0$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)} \quad \sin(t) \neq 0$$

$$\sec(t) = \frac{1}{\cos(t)} \quad \cos(t) \neq 0$$

$$\csc(t) = \frac{1}{\sin(t)} \quad \sin(t) \neq 0$$



Use this mnemonic to memorize the Pythagorean and Reciprocal identities

1. Verify $\csc(\theta) \cos(\theta) \tan(\theta) = 1$

2. Fill in the blanks and complete the following Pythagorean identities.

(a) $\sin^2(x) + \cos^2(x) = \underline{\hspace{2cm}}$

(c) $\sec^2(x) - 1 = \underline{\hspace{2cm}}$

(b) $1 - \sin^2(x) = \underline{\hspace{2cm}}$

(d) $1 + \tan^2(x) = \underline{\hspace{2cm}}$

3. Verify the identity

$$\tan^2(\theta) \left(\frac{1}{\sec(\theta) - 1} + \frac{1}{\sec(\theta) + 1} \right) = 2 \sec(\theta).$$

4. Verify the following trigonometric identity:

$$\csc(x) = \frac{\tan(x)}{\sec(x) - \cos(x)}$$

5. Find **all** solutions to the following **equation**. Express your answer **in radians**.

$$2\sin^2(\theta) = 3\sin(\theta) - 1$$

Example Videos:

- https://mediahub.ku.edu/media/t/1_9y12o54v